Block Model for Growth of Steam-Heated Zone in Oil-Bearing Naturally Fractured Carbonate Reservoirs

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Summary
A model for steam injection into naturally fractured carbonate reservoirs provides a method of calculating the volume of heated formation for a given amount of steam, provided that the assumptions of the model are satisfied. The model represents the fractured reservoir by a system of blocks, dependent on fracture spacing and dimensions. The amount of oil mobilized can be estimated from prior measurements of oil saturation and laboratory tests of oil-displacement efficiency. Results comparable to observations are obtained. In addition, the temperature distribution along the direction of propagation in the reservoir may also be estimated, providing a guide to variation in the local oil mobility. These results can assist in evaluating the effectiveness of steam injection in the recovery of oil from naturally fractured formations.

Introduction
The numerous heavy-oil carbonate reservoirs throughout the world provide significant targets for thermal recovery (Iqbal et al. 2009; Penney et al. 2005). Many of these reservoirs are characterized by the presence of fractures within large volumes of oil-bearing low-permeability rock. The development of models to represent such systems accurately depends on reliable information about fracture spacing and about the variability of rock porosity. The latter may be characterized by pore sizes ranging from micron sized to vugs that are easily visible in rock samples (Iqbal et al. 2009). The portion of rock between fractures may be a series of pieces of various sizes and shapes with variable spacing between the fractures. The challenge for a model, then, is to represent the reservoir by physical parameters that describe the variable rock properties and the relevant flow capacity. Steam injection in such cases has been complicated by channeling of injected steam and condensate through fractures, bypassing large portions of the reservoir but heating the oil by conduction from the fracture surface (Penney et al. 2007). If there is sufficient matrix permeability and porosity, portions of the oil may also be heated and displaced by flow of steam through the permeable portions of the rock. The heated oil can then be displaced to a location in the reservoir where it can be recovered. At the same time, the oil may undergo viscosity reduction, thermal expansion, and chemical changes, along with alterations of the rock (Mollaei and Maini 2010). It would be useful, therefore, to have a method of estimating the amount of steam needed to heat and mobilize the oil present in a given volume of reservoir. Information on the methods of displacement and recovery of the heated oil can be provided by laboratory and field studies on the reservoir by samples from the field of interest.

Model Assumptions
1. The reservoir contains vertical and horizontal fractures that are highly permeable to fluid flow.
2. The flow capacity of the reservoir consists of the flow capacity of the fracture system and any permeable rock within the fracture system.
3. The reservoir, with its fractures, may be considered an accumulation of blocks. Heat from steam will then flow by conduction from all sides of the blocks into the center of the blocks (Fig. 1). If the blocks are permeable to the steam and condensate, additional heating and oil displacement will take place. If permeability is small, it is assumed that there are sufficient paths for the heated oil to flow into the fracture system by gravity drainage or by whatever local pressure gradients develop. For these cases, the applicability of the model will depend on accurate geological information about fracture spacing, length, and height. This heat-flow model uses essentially the same heat-balance approach as that of Marx and Langenheim (1959) for controlled injection of steam into the reservoir, which has been used for other reservoir conditions (Closmann 1967, 1968).
4. Oil displacement will be determined essentially by the oil-saturation change in the steam-heated volume. Oil-recovery efficiency will depend on temperature and will also be a function of the types of porosity and permeability and the oil distribution for a given rock. It will also depend on the effects of gravity override. These factors are usually determined by geologic and petrophysical studies and laboratory tests of cores, as well as field experience for a given reservoir. It will be important to evaluate the distribution of steam flow between fractures and permeable rock.

The model to be described in the following section does not account for the many chemical and physical effects that have been described (Mollaei and Maini 2010). These could be included in a more comprehensive model, after more field experience is gained in naturally fractured reservoirs. Also, the quantity of oil produced may well be smaller than the oil mobilized in this model, because of sweep-efficiency variations.

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were characterized by the same quantity of permeable rock, thus averaging rock properties for this calculation.

5. The heat conduction into the blocks is based on conduction from the surface of a sphere with the same volume and properties as the average block (Carslaw and Jaeger 1959). This formulation will be used for all block shapes. For cubical blocks, the use of the sphere will involve a heat-flow surface 81% of that of the cube. Nevertheless, the spherical surface area will be used, as it is consistent with the interior heat-flow profiles adopted in this model. For flat blocks with at least one dimension shorter that the others, the heat flow into the block could be considerably more effective along the shortest dimension than along the others.

For cases in which the horizontal fracture spacing is known but the vertical spacing is not apparent, the engineer can assume vertical and lengthwise block dimensions that appear consistent with the reservoir. The only limitation on the resulting equivalent spherical block is that it fit within the vertical limits of the swept zone.

6. Heat flow into caprock and base rock is by 1D heat conduction from the steam-heated formation. The effective reservoir thickness may be adjusted to account for gravity overlay or the presence of low or high permeability streaks.

7. The contained hydrocarbons have sufficient access to the fracture system to respond to the pressure gradients applied during steam injection or to drain by gravity drainage when heated.

8. Steam may be injected at the rate specified and the time duration specified, without being affected by pressure buildup or the development of altered rock and oil properties. The model applies to both vertical and horizontal wells.

Derivation of Conductive Heat Flow Into Blocks
The temperature gradient in a heated sphere is calculated from temperature distributions plotted by Carslaw and Jaeger (1959). These values are shown on the semilog plot of Fig. 2. The nature of the gradient changes over time. For early times less than or equal to a dimensionless time $t_D = 0.05$, an inverse square-root-of-time dependence, shown in Fig. 2, is used to represent the Carslaw and Jaeger results:

$$\frac{dT_D}{D_D} = -\beta t_D^{0.5}, \text{ ................................. (1)}$$

where $\beta$ has the value 0.3683 and dimensionless time $t_D = \frac{r_b t}{r_h^2}$.

For longer dimensionless times (greater than 0.05), the straight-line portion of the Carslaw and Jaeger results shown in Fig. 2 is represented by the equation

$$\frac{dT_D}{D_D} = -a e^{-b t_D}, \text{ ................................. (2)}$$

In the following application, $a = 2.684$ and $b = 9.767$. The approximate value of dimensionless time at which these two gradient forms intersect (contact time) is $t_D = 0.05$. In generating the constants to represent the results of Carslaw and Jaeger, it was also important to obtain values that would provide consistent overall results in temperature calculations. In particular, as time increases, the block temperature should approach the steam temperature. This constraint also determines the value of the parameter $\Theta$ (to be later defined) as 0.33333 and was consistent with the values of the parameters $a$, $b$, and $\beta$.

Basic Heat-Flow Equation During Steam Injection
The input data, which include fracture dimensions, are indicated in Appendix A, in which the block sizes and parameters are calculated.

The basic heat-balance equation, following a procedure similar to that of Marx and Langenheim (1959), is as follows:

$$q_s H_s = f M_s \Delta T \frac{dv_i}{d\tau} + (1-f)k_B \Delta T \frac{A_B}{r_B V_B} \int_0^{G(t-\tau)} \frac{dv_i}{d\tau} d\tau + \frac{2k_o \Delta T}{\ln \frac{r_B}{r_h}} \int_0^{G(t-\tau)} \frac{dv_i}{d\tau} d\tau, \text{ ................................. (3)}$$

where $f$ is the fraction of formation heat that flows into permeable portions of the blocks and $G(t)$ accounts for the temperature gradient inside an individual block surface. The first term on the right side of Eq. 3 has the same form as the original ML relation. This portion of the rock will be at steam temperature. The other portions of the rock will be heated by conduction from the block surfaces, their temperatures approaching that of steam. On the basis of published information (Iqbal et al. 2009; Mollaei and Maini 2010), it is assumed that heating by conduction from the block surfaces will be the majority process for heating the reservoir, and $f$ will usually be zero. However, for reservoirs in which there is information on flow through permeable portions, the value of $f$ can be set to its best estimate. (In the limit $f = 1$, the equation reduces to the ML formulation.)

The second term on the right side of Eq. 3 contains the ratio of total heated volume $V_i$ to block volume $V_B$ and, thus, reflects the number of blocks being heated. The third term accounts for heat loss to overburden and underburden and is the same as that in the original ML formulation.
The solution for the steam-heated volume $V_s$ as a function of dimensionless time $t_D$ is developed in Appendix B and makes use of the Stehfest algorithm (Stehfest 1970) for inversion of Laplace transforms. Laplace-transform inversions may also be performed with MATLAB®, Mathematica®, or other math-type programs. Results obtained in this work are presented in tabular form in Table 1. They are also plotted in Fig. 3. Values in Table 1 are numerical values of the dimensionless function $V(\lambda, t_D)$, which has the functional form of Eq. 1 when dimensionless time $t_D$ is less than or equal to 0.05 and has the form of Eq. 2 when dimensionless time is greater than 0.05.

**Solution**

The solution for the steam-heated volume $V_s$ as a function of dimensionless time is developed in Appendix B and makes use of the Stehfest algorithm (Stehfest 1970) for inversion of Laplace transforms. Laplace-transform inversions may also be performed with MATLAB®, Mathematica®, or other math-type programs. Results obtained in this work are presented in tabular form in Table 1. They are also plotted in Fig. 3. Values in Table 1 are numerical values of the dimensionless function $V(\lambda, t_D)$, which has the functional form of Eq. 1 when dimensionless time $t_D$ is less than or equal to 0.05 and has the form of Eq. 2 when dimensionless time is greater than 0.05.

In Eq. 3, the function $G(t)$ has the functional form of Eq. 1 when dimensionless time $t_D$ is less than or equal to 0.05 and has the form of Eq. 2 when dimensionless time is greater than 0.05.

**Applications and Discussion**

The following examples are intended to demonstrate the application of this model to possible values of reservoir variables. More-exact and -appropriate values should be supplied by engineers and geologists familiar with the different formations involved. Experience with applications may enable improvements to be made in the model.

**Example 1.** Estimation of the size of the heated zone and temperature distribution after 1 and 2 years, and the mobilized oil/steam (O/S) ratio after 1 year. Conditions are similar to those in Oman (Penney et al. 2007).

- Fracture (block) dimensions (ft): 50 × 50 × 50
- Reservoir temperature: 100°F
- Steam temperature: 538°F
- Steam-injection rate: 2.2 × 10^6 lbm/D
- Enthalpy change: 995 Btu/lbm with 80% quality
- Physical properties may be estimated by reference to SPE Monograph (Prats 1982)
  - Block thermal diffusivity: 0.63 ft^2/D
  - Block thermal conductivity: 22 Btu/F/ft/D
  - Formation thermal diffusivity: 0.63 ft^2/D
  - Formation thermal conductivity: 22 Btu/F/ft/D
  - Formation volumetric heat capacity: 35 Btu/ft^3/F
  - Reservoir thickness: 120 ft
  - Porosity: 26%
TABLE 2—ADVANCE OF HOT ZONE—50 FT BLOCK MODEL—SINGLE VERTICAL WELL—RADIAL FLOW—STEAM TEMPERATURE 538°F

<table>
<thead>
<tr>
<th>Dim. Time</th>
<th>Time (days)</th>
<th>Total Dim. Volume</th>
<th>Position from Injector (ft)</th>
<th>Dim. Time Diff. From 0.239 (1 year)</th>
<th>Dim. Time Diff. From 0.478 (2 years)</th>
<th>Temp. Profile (F) at Dim. Time</th>
<th>Temp. Profile (F) at Dim. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1.53</td>
<td>0.00746</td>
<td>91</td>
<td>0.238</td>
<td>0.477</td>
<td>503</td>
<td>535</td>
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<td>0.002</td>
<td>3.05</td>
<td>0.01054</td>
<td>109</td>
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<td>0.476</td>
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<td>535</td>
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<tr>
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<td>0.229</td>
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<td>534</td>
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<td>193</td>
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<td>495</td>
<td>534</td>
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<tr>
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<td>255</td>
<td>0.179</td>
<td>0.418</td>
<td>475</td>
<td>532</td>
</tr>
<tr>
<td>0.075</td>
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<td>0.06466</td>
<td>269</td>
<td>0.164</td>
<td>0.403</td>
<td>465</td>
<td>531</td>
</tr>
<tr>
<td>0.100</td>
<td>153</td>
<td>0.07520</td>
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<td>445</td>
<td>529</td>
</tr>
<tr>
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<td>358</td>
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<td>0.278</td>
<td>291</td>
<td>514</td>
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<tr>
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<td>0.12355</td>
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<td>0.253</td>
<td>215</td>
<td>507</td>
</tr>
<tr>
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<td>0.000</td>
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<td>100</td>
<td>503</td>
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<tr>
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<td>461</td>
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<td>440</td>
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<td>0.118</td>
<td>369</td>
<td>333</td>
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<tr>
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<td></td>
<td>0.078</td>
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<td>0.058</td>
<td>333</td>
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<tr>
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<td></td>
<td>0.018</td>
<td>230</td>
<td>230</td>
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<tr>
<td>0.470</td>
<td>718</td>
<td>0.21164</td>
<td>487</td>
<td></td>
<td>0.008</td>
<td>187</td>
<td>187</td>
</tr>
<tr>
<td>0.478</td>
<td>730</td>
<td>0.21411</td>
<td>490</td>
<td></td>
<td>0.000</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

- Initial oil saturation: $S_{oi} = 0.90$
- $f = 0$
- Residual oil saturation: $S_{or} = 0.20$, an estimate based on studies of steamdrive residual oil values in sandstones (Clossmann and Seba 1983)
- Recovery efficiency: 0.50 (see Penney et al. 2005)

**Solution**
- Spherical block radius = $0.62035(50) = 31.02$ ft
- $\lambda \approx (3/2)(120)/31.02 = 5.80$

(For a first estimate, assume the ratios of thermal conductivity and diffusivity are equal to unity.) Dimensionless volume as a function of time, with $\lambda = 5.80$, was calculated as indicated in Appendix B (Eqs. B-15, B-21, and B-22) by means of the Stehfest algorithm and is indicated in **Table 2**. (The first four rows in Column 1 were calculated with Eq. B-15 for early-time solutions.) To calculate the extent of heating along the direction of propagation, assume that the heated zone grows in a radial pattern of thickness $120$ ft from a single vertical well.

To estimate the O/S ratio after 1 year, use the residual oil saturation of 0.20 and the 50% recovery efficiency suggested. [The relatively low temperatures in blocks near the advancing heat front (Table 2) suggest a possible range in recovery in the heated zone, because of possible variations in oil viscosity as well as other factors]. In Table 2, radial distance to the front is 380 ft. In this case,

$$O/S = \pi(380)^2(120)(0.26)(0.70)(0.50)(2.8317 \times 10^{-2} \times 10^2)(1000)/(2.2 \times 10^8(0.45359)(365)) = 0.39 \text{ m}^3/\text{tonne}$$

This result is within the range reported (0.1– 0.4 m$^3$/tonne; Penney et al. 2007), but many uncertainties in the various parameters could affect it, as well as effects of gravity override and changes taking place during the steamflood. The chief use of this method may be to indicate the potential expected recovery, as well as the ability to indicate the sensitivity of results to the variations in input data.

In this example, the temperature distribution in the blocks along the flow direction after 1 and 2 years is desired. As indicated by the method in Appendix C, the time of heating the blocks at a particular position along the flow path after 1 year is calculated as the difference between 1 year and the arrival time of the heat front at that particular position. The dimensionless arrival time is given in Column 1 of Table 2. The duration of heating at each position after 1 year of injection is then the difference between the total 1-year dimensionless-time value (0.239) and the arrival time of the heat front (Column 1 of Table 2) and is given in Column 5 of Table 2. These values are substituted in Eq. C-3a for times less than 0.05 and Eq. C-3b for times greater than 0.05, and the results are entered into Column 7 of Table 2. A similar procedure was followed for the 2-year distribution, with results in Columns 6 and 8. The temperature distributions are plotted in **Fig. 4**. In the figure, it can be seen that a significant portion of the temperature in the heated zone is below the steam temperature (538°F).

This result can be compared with that predicted by the ML equation for steam injection into a permeable reservoir, with the same basic parameters as in the preceding. The following ML equation is given by Burger et al. (1985), who tabulate the dimensionless volume function.

![Fig. 4—Temperature distributions in the steam-heated zone after 1 year and 2 years of steam injection (ML at 1 year).](image-url)
Table 3—Advance of Hot Zone—20×20×10-ft Block Model, With f = 0.2 and f = 0.0—Single Vertical Well—Radial Flow—Steam Temperature 456°F

<table>
<thead>
<tr>
<th>Dim. Time</th>
<th>Time Before Time</th>
<th>Dim. Volume</th>
<th>4(02) at Column 1 Time</th>
<th>Dim. Volume</th>
<th>02 at Column 1 Time</th>
<th>Dist. From Injector (ft)</th>
<th>Temp. at Column 6 (°F)</th>
<th>Temp. at Column 8 (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>14</td>
<td>2.535</td>
<td>0.03378</td>
<td>0.03746</td>
<td>20.2</td>
<td>21.3</td>
<td>456</td>
<td>456</td>
</tr>
<tr>
<td>0.80</td>
<td>111</td>
<td>1.835</td>
<td>0.16728</td>
<td>0.16970</td>
<td>45.0</td>
<td>45.3</td>
<td>456</td>
<td>456</td>
</tr>
<tr>
<td>1.60</td>
<td>222</td>
<td>1.035</td>
<td>0.30614</td>
<td>0.30820</td>
<td>60.9</td>
<td>61.1</td>
<td>456</td>
<td>456</td>
</tr>
<tr>
<td>2.00</td>
<td>277</td>
<td>0.635</td>
<td>0.37209</td>
<td>0.37401</td>
<td>67.1</td>
<td>67.3</td>
<td>455</td>
<td>455</td>
</tr>
<tr>
<td>2.20</td>
<td>305</td>
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<td>0.40434</td>
<td>0.40621</td>
<td>70.0</td>
<td>70.2</td>
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<td>2.40</td>
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<td>0.45192</td>
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<td>0.085</td>
<td>0.45976</td>
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<td>74.8</td>
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<td>75.7</td>
<td>75.9</td>
<td>49</td>
<td>49</td>
</tr>
</tbody>
</table>

Example 2. Partial steam flow through the interior of the block, with conditions similar to those in Mollaei et al. (2007).

- Fracture (block) dimensions (ft): 20×20×10
- Reservoir temperature: 49°F
- A portion of steam flows through porous rock; assume a value of f = 0.2 (value chosen arbitrarily to demonstrate the method)
- Steam temperature: 456°F
- Steam-injection rate: 7.0×104 lbm/D (200 B/D)
- Enthalpy change: 1,034 Btu/lbm with 80% quality
- Block thermal diffusivity: 0.7 ft²/D
- Block thermal conductivity: 20 Btu/ft/°F
- Block volumetric heat capacity: 35 Btu/ft³
- Formation thermal diffusivity: 0.7 ft²/D
- Formation thermal conductivity: 23 Btu/ft²/D
- Reservoir thickness: 100 ft
- Porosity: 0.15
- Initial oil saturation: Soi = 0.85
- Residual oil saturation: Sor = 0.20
- Estimated recovery: one-half of available saturation change
- Volumetric heat capacity of steam-invaded rock: M = 33 Btu/ft³
- Estimated volume of steam injected: M = 33 Btu/ft³

Solution. Spherical block radius:

\[
 r_0 = 0.062035(20)(20)(10) = 6.2035 \text{ ft}
\]

Dimensionless heated volume, calculated as indicated in Appendix B (Eq. B-13 and inversion of Eq. B-18 by means of the Stehfest algorithm), is shown as a function of time in Table 3. The temperatures in the table are calculated by Eqs. C-3a and C-3b in Appendix C, with f = 0.2 and with f = 0 for a single vertical well. In Table 3, radial distance to the front is 75.9 ft for the f = 0 case and 75.7 ft for the f = 0.2 case. A slight adjustment in the value of the ratio M/M (Prats 1982) was necessary to limit maximum temperature to 456°F. Then,

\[
 O/S = (75.75)^2(100)(0.25)(0.325)(2.831710^{-2})(1.000)/[(70,000)(0.45359)(365)] = 0.36 \text{ m³/tonne}.
\]

This result is within the range reported (Mollaei et al. 2007). The temperature values in Columns 8 and 9 of Table 3 show that a significant part of the heated zone is at steam temperature (456°F). In Columns 6 and 7, the advance of the heat front for the conductive-heat-flow case (f = 0) is slightly farther than that of the f = 0.2 case for long times, because individual blocks do not accept as much heat by the advancing steam. Hence, the front moves farther. Temperature values calculated with Eqs. C-3a and C-3b are shown in Table 3 and plotted in Fig. 5. The temperature
values at block heating times below 0.05 for the \( f = 0.2 \) case are lower than those of the \( f = 0 \) case, deviating slightly from the long-time trend and eventually converging to the formation temperature at the position of the heat front. Further study of target reservoirs should improve the knowledge of petrophysical parameters and the development of this model as well as others.

**Discussion**

The results in the preceding examples indicate a model that is useful for estimating the expected recovery when steam is injected into a reservoir with natural fractures, provided that the assumptions of the model are satisfied. In this respect, the values of the various reservoir parameters will be critical and will depend on the degree of homogeneity of the reservoir. With various block shapes, a model can be useful in determining the sensitivity of results to values chosen for specific reservoir parameters. Because the petrophysical properties of fractured reservoirs can be very uncertain, the actual performance of a project could differ considerably from that predicted. Furthermore, the possible alterations in chemical and physical properties can change the behavior of the system during injection. Such factors as flow capacity and pressure drop could be affected and would be important. As more experience is gained, altered properties could be incorporated in later applications of the model.

Other factors that can be very important are the effects of gravity override and the presence of permeable streaks. Smaller layer thicknesses could considerably modify the heat loss, and lower the value of the O/S ratio. Superimposed on all these factors is the effect of inhomogeneities. More experience with fractured reservoirs in general and with specific fields could make this model more useful.

**Conclusions**

A block model has been developed to calculate the size of the steam-heated zone generated by steam injection into a naturally fractured carbonate reservoir. To determine the applicability of this model to a particular reservoir, it is recommended that studies be made of the fracture distribution and characteristics, and that tests be made of oil-recovery efficiency on samples of the formation. To the extent that the assumptions of the model are satisfied, the following conclusions can be drawn:

1. A model has been developed to estimate the size of the heated zone during steam injection in a formation with multiple fractures.
2. The temperature distribution in the model can be estimated, providing a means of evaluating the effectiveness of heating and of possible related changes in the local displacement efficiency of oil.
3. The O/S ratio of mobilized oil can be predicted if representative values of initial oil saturation and residual oil saturation can be obtained from preliminary measurements on samples from the reservoir.

**Nomenclature**

\[ a = \text{dimensionless coefficient in equation for temperature gradient in spheres} \]

\[ a_{ML} = m(f \gamma \lambda) \]

\[ A_g = \text{surface area of block, ft}^2 \text{[m}^2] \]

\[ b = \text{coefficient of dimensionless time in exponent of heat-conduction equation for sphere} \]

\[ f = \text{fraction of permeable reservoir rock} \]

\[ F = f \gamma \lambda + (1 - f) \lambda \Theta \]

\[ g(s) = \text{Laplace transform of } G(t_d) \]

\[ G(t_d) = \text{dimensionless temperature-gradient function at block surface} \]

\[ h = \text{reservoir thickness, ft [m]} \]

\[ H_s = \text{enthalpy of steam above reservoir temperature, Btu/lbm} \text{[kJ/kg]} \]

\[ k_b = \text{thermal conductivity of block, Btu-ft/(D-ft}^2\text{-C}) \text{[W/(m-K)]} \]

\[ k_o = \text{thermal conductivity of overburden, Btu-ft/(D-ft}^2\text{-C}) \text{[W/(m-K)]} \]

\[ L_x = \text{block width in x-direction, ft [m]} \]

\[ L_y = \text{block length in y-direction, ft [m]} \]

\[ L_z = \text{block height in z-direction, ft [m]} \]

\[ m = 1 + (1 - f) \beta \sqrt{\pi} \]

\[ M_p = \text{block volumetric heat capacity, Btu/ft}^3/F \text{[kJ/m}^3\text{K]} \]

\[ M_v = \text{volumetric heat capacity of overburden, Btu/ft}^3/F \text{[kJ/m}^3\text{K]} \]

\[ r = \text{radial variable inside block, ft [m]} \]

\[ r_e = \text{equivalent spherical-block radius, ft [m]} \]

\[ r_d = \text{dimensionless radial distance, ft [m]} \]

\[ S_i = \text{initial oil saturation, fraction} \]

\[ S_o = \text{residual oil saturation, fraction} \]

\[ t = \text{time, days} \]

\[ t_{BD} = \text{change time; dimensionless time for change in heat-flow function in interior of block} = 0.05 \]

\[ t_{Di} = \text{dimensionless time in heat-conduction equations} = a_{ML} / r_e^2 \]

\[ t_{DF} = \text{dimensionless time of contact of ith block by heat front} \]

\[ t_{DM} = \text{dimensionless time in ML model} \]

\[ T_B = \text{block temperature, } ^\circ F \text{[C]} \]

\[ T_T = \text{dimensionless block temperature, } (T_B - T_p)/(T_T - T_p) \]

\[ T_r = \text{original reservoir temperature, } ^\circ F \text{[C]} \]

\[ V = \text{dimensionless steam-heated volume} \]

\[ V_B = \text{volume of block, ft}^3 \text{[m}^3] \]

\[ V_r = \text{steam-heated volume, ft}^3 \text{[m}^3] \]

\[ V_{MB} = \text{steam-zone volume in ML model, ft}^3 \text{[m}^3] \]

\[ \beta = \text{coefficient of inverse of square-root of dimensionless time in block rate function} \]

\[ \gamma = M_v / (3 M_p) \]

\[ \Delta T = T_B - T_p, ^\circ F \text{[C]} \]

\[ \Theta = 2\beta \sqrt{\pi} + (a/b) \exp(-bt_{Di}) \]

\[ \lambda = 3kbh/\sqrt{\pi} / (2k\sqrt{a/b}) \]

\[ \tau = \text{variable time in integration, days} \]

\[ \tau_{Di} = \text{variable dimensionless time in integration} \]

\[ \Phi = \lambda (\Theta - a / b) \]

\[ \Omega = F - (1 - f) \lambda a / b \]

**Constants**

\[ a = 2.684 \]

\[ b = 9.767 \]

\[ a / b = 0.274803 \]

\[ t_{Di} = 0.05 \]

\[ \beta = 0.3683 \]

\[ \Theta = 0.33333 \]

\[ \Phi = 0.05853 \lambda \]

**Acknowledgment**

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**References**


Appendix A—Fracture and Block Dimensions

$L_h = $ horizontal fracture spacing $= $ block width.

$L_l = $ horizontal fracture length $= $ block length.

$L_b = $ fracture height $= $ block height.

Dimensions of typical block: $L_h, L_l, L_b$

Block volume: $V_B = L_hL_lL_b$

Radius of sphere of equal volume: $r_B = (3/4)V_B/\pi^{1/3} = 0.620350V_B^{1/3}$

Surface area of sphere of equal volume: $A_B = (4\pi)^{1/3}(3V_B)^{2/3} = 4.835976V_B^{2/3}$

$A_B = 3V_B/r_B$

Appendix B—Solution of the Basic Heat-Flow Equation for the Steam-Heated Volume

Convert Eq. 3 to a form containing dimensionless time $t_D = z_Bt/r_B^2$:

$$qH_i = f M_l\Delta T \frac{dV}{dt_B} \left| \begin{array}{c}
+ (1 - f) k_B \Delta T \int_0^\infty \left( \frac{A_B}{r_B} \int_0^{t_D} G(t_D) - t_D \right) \frac{dV}{dt_D} dt_D \\
+ \frac{2k_B\Delta T \sqrt{2} \rho_B}{h\pi\nu_{\infty}} \int_0^\infty \frac{dV}{dt_D} dt_D - t_D \right.
\end{array} \right. \quad \text{(B-1)}$$

The dimensionless block conductive-heat-loss function $G(t_D)$ (Carslaw and Jaeger 1959) is defined as follows (Churchill 1973):

$$G(t_D) = \beta t_D^{1.5}, \quad 0 \leq t_D \leq t_D_D \quad \text{(B-2)}$$

$$= a \exp(-b t_D), \quad t_D > t_D_D \quad \text{(B-3)}$$

where the function change time $t_D_D = 0.05$ in calculations.

Solution of Eq. B-1. Define the Laplace transform as follows:

$$\mathcal{L}_s \left[ \int_0^\infty e^{-st_B}t_B \frac{dV}{dt_B} \right] = \mathcal{L}_s \left[ \int_0^\infty e^{-st_B} \frac{dV}{dt_B} \right] \quad \text{(B-4)}$$

Then solve for $\mathcal{L}_s$ from Eq. B-1, obtaining

$$\mathcal{L}_s = \frac{qH_i h T_B}{2k_B \Delta T} \sqrt{\frac{\rho_B}{2\pi g}} s^{-3/2} \left[ f \gamma \lambda \sqrt{s} + (1 - f) \lambda \sqrt{s} g(s) + 1 \right]^{-1}, \quad \text{(B-5)}$$

where

$$\lambda = \frac{3k_B h}{2k_B r_B} \sqrt{\frac{\rho_B}{2\pi g}} \quad \text{(B-6a)}$$

$$\gamma = \frac{\rho_B C_1}{3p_B C_B} \quad \text{(B-6b)}$$

and $g(s)$ is calculated from $G(t_D)$ as follows (Churchill 1973):

$$g(s) = s \int_0^\infty \{ G(t_D) \mathcal{L}_s \} dt_D \quad \text{(B-7)}$$

where $\mathcal{L}_s$ indicates the Laplace transformation of the quantity in brackets. From Eq. B-5, we obtain the transform of a dimensionless steam-heated volume $V$:

$$V = s^{-3/2} \left[ f \gamma \lambda \sqrt{s} + (1 - f) \lambda \sqrt{s} g(s) + 1 \right]^{-1} \quad \text{(B-8)}$$

For Times Less Than or Equal to $t_D_D$, From Eq. B-2,

$$g(s) = \beta \sqrt{s} \quad \text{(B-9)}$$

and

$$V = s^{-3/2} \left[ f \gamma \lambda \sqrt{s} + (1 - f) \beta \lambda \sqrt{s} + 1 \right]^{-1} \quad \text{(B-10)}$$

The solution for the dimensionless steam-heated volume is (Erdeőlyi et al. 1984)

$$V = \frac{f \gamma \lambda}{m^2} \left[ \frac{2}{\sqrt{\pi}} a_{ML} \sqrt{t_D} + \exp(a_{ML}^2 t_D) \text{erfc}(a_{ML} \sqrt{t_D}) - 1 \right], \quad \text{(B-11)}$$

where

$$m = 1 + (1 - f) \beta \lambda \sqrt{\pi} \quad \text{(B-12a)}$$

$$a_{ML} = m/(f \gamma \lambda) \quad \text{(B-12b)}$$

The ML function in Eq. B-11 can be approximated by a simpler form (Burger et al. 1985), replacing Eq. B-11 by

$$V = \frac{f \gamma \lambda}{m^2} \left[ 1 + 0.85 a_{ML} \sqrt{t_D} \right] \quad \text{(B-13)}$$

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For the case \( f = 0 \),
\[
\nabla = x^{-3/2}(1 + \beta x \sqrt{\pi})^{-1}
\]
\[\text{for } \text{Times Greater Than } t_{D},\]
\[
V = 2\sqrt{t_{D}/\pi} \frac{2b(1 + \beta x \sqrt{\pi})^{-1}}{1 + \beta x \sqrt{\pi}}
\]
\[\text{For } t_{D} - t_{D_{i}} > 0.05,\]
\[
V = \frac{s + b}{s^{3/2}(x s^{3/2} + s + \beta b \sqrt{x + b})} \frac{1}{s^{3/2} + x + \beta b \sqrt{x + b}}
\]

\[\Theta = 2\beta \sqrt{t_{D}} \frac{x \sqrt{\pi}}{b} \exp(-bt_{D})\]
\[\text{Eq. B-5 becomes}\]
\[
V = \frac{s + b}{s^{3/2}(\Theta s^{3/2} + s + \beta b \Theta \sqrt{s + b})}
\]
\[\text{For } t_{D} - t_{D_{i}} \leq 0.05,\]
\[
Q = f M_{c} \Delta T V_{B} \frac{\frac{2}{(1 + \beta t_{D} - t_{D_{i}})}}{1 + 0.85 \frac{\alpha_{ML} \sqrt{t_{D} - t_{D_{i}}}}{\beta}}
\]
\[
+ (1 - f) \frac{k_{B} A_{B} r_{B} \Delta T}{2b} \left( \int_{t_{D_{i}}}^{t_{D}} \frac{d\tau}{\sqrt{\tau - \tau_{D_{i}}}} \right)
\]
\[\text{For } t_{D} - t_{D_{i}} \leq 0.05,\]
\[
Q = f M_{c} \Delta T V_{B} \frac{\frac{2}{(1 + \beta t_{D} - t_{D_{i}})}}{1 + 0.85 \frac{\alpha_{ML} \sqrt{t_{D} - t_{D_{i}}}}{\beta}}
\]
\[
+ (1 - f) \frac{k_{B} A_{B} r_{B} \Delta T}{2b} \left( \int_{t_{D_{i}}}^{t_{D}} \frac{d\tau}{\sqrt{\tau - \tau_{D_{i}}}} \right)
\]
\[\text{The heat is determined from the time integral of the heat-flow rate into the block, as follows:}\]
\[\text{For the heat flow in the permeable part of the block at early time, an approximation that uses the ML function (see Eq. B-13) can be used. (It is assumed that all blocks share the overall rock properties equally.)}\]
\[\text{For } t_{D} - t_{D_{i}} \leq 0.05,\]
\[
T_{B_{i}} = T_{f} + f M_{c} \Delta T \frac{\frac{2}{(1 + \beta t_{D} - t_{D_{i}})}}{1 + 0.85 \frac{\alpha_{ML} \sqrt{t_{D} - t_{D_{i}}}}{\beta}}
\]
\[\text{For } t_{D} - t_{D_{i}} > 0.05,\]
\[
T_{B_{i}} = T_{f} + f M_{c} \Delta T \frac{2}{1 + \beta \sqrt{t_{D}} - \beta \sqrt{t_{D_{i}}}}
\]
\[\text{SI Metric Conversion Factors}\]
\[
\begin{array}{ll}
\text{bbl} & \times 1.589 873 \quad \text{E-01} = \text{m}^{3} \\
\text{Btu/ft}^{2}/\text{F} & \times 6.706 611 \quad \text{E+01} = \text{kJ/m}^{3}/\text{K} \\
\text{ft} & \times 3.048 \quad \text{E-01} = \text{m} \\
\text{ft}^{2} & \times 9.290 304 \quad \text{E-02} = \text{m}^{2} \\
\text{ft}^{3} & \times 2.831 685 \quad \text{E-02} = \text{m}^{3} \\
\text{°F} & \times 32/1.8 \quad \text{E-02} = \text{°C} \\
\text{lbm} & \times 4.535 924 \quad \text{E-01} = \text{kg} \\
\text{tonne} & \times 1.0 \quad \text{E+03} = \text{kg}
\end{array}
\]
\[\text{Conversion factor is exact.}\]
Philip J. Closmann worked for 33 years with the Shell Development Company on thermal recovery methods for heavy oil and oil shale. Following retirement, he taught thermal recovery at the University of Houston and has conducted one-week industry courses and two-day workshops on steamflooding for SPE. Closmann holds a BE degree in chemical engineering from Tulane University, an SM degree in chemical engineering from Massachusetts Institute of Technology, an MS degree in physics from California Institute of Technology, and a PhD degree in physics from Rice Institute. He has served on the SPE editorial committee and on the Technology Today Series committee, serving one year as chairman. Closmann was a recipient of the 2004 SPE/Department of Energy Improved Oil Recovery award.