A New Analytical Method for Analyzing Linear Flow in Tight/Shale Gas Reservoirs: Constant-Flowing-Pressure Boundary Condition

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Summary

Many tight/shale gas wells exhibit linear flow, which can last for several years. Linear flow can be analyzed using a square-root-of-time plot, a plot of rate-normalized pressure vs. the square root of time. Linear flow appears as a straight line on this plot, and the slope of this line can be used to calculate the product of fracture half-length and the square root of permeability.

In this paper, linear flow from a fractured well in a tight/shale gas reservoir under a constant-flowing-pressure constraint is studied. It is shown that the slope of the square-root-of-time plot results in an overestimation of fracture half-length, if permeability is known. The degree of this overestimation is influenced by initial pressure, flowing pressure, and formation compressibility. An analytical method is presented to correct the slope of the square-root-of-time plot to improve the overestimation of fracture half-length. The method is validated using a number of numerically simulated cases. As expected, the square-root-of-time plots for these simulated cases appear as a straight line during linear flow for constant flowing pressure. It is found that the newly developed analytical method results in a more reliable estimate of fracture half-length, if permeability is known. Our approach, which is fully analytical, results in an improvement in linear-flow analysis over previously presented methods. Finally, the application of this method to multifractured horizontal wells is discussed and the method is applied to three field examples.

Introduction

The dominant flow regime observed in most fractured tight/shale gas wells is linear flow, which may continue for several years. The square-root-of-time plot, normalized pressure vs. square root of time, is probably the most important plot for analyzing linear flow (Anderson et al. 2010). Linear flow appears as a straight line on the square-root-of-time plot. The slope of this line can be used to calculate the product of fracture half-length and the square root of permeability. This means that the fracture half-length can be derived from linear-flow analysis, given that the permeability is known. The y-intercept on the square-root-of-time plot represents an apparent skin.

It is reported in the literature that using the slope of the square-root-of-time plot leads to an overestimation of \( {x_f \sqrt{k}} \) (Ibrahim and Wattenbarger 2005, 2006; Nobakht et al. 2010), where \( x_f \) is the fracture half-length and \( k \) is the permeability. Ibrahim and Wattenbarger (2005, 2006) introduced a correction factor that corrects the slope of the square-root-of-time plot and improves the \( {x_f \sqrt{k}} \) values obtained from the square-root-of-time plot. They developed the following empirical equation to estimate the correction factor under a constant-flowing-pressure condition:

\[
fc_{PR} = 1 - 0.0852D_{ij} - 0.0857D_{ij}^2, \quad \text{................. (1)}
\]

where \( fc_{PR} \) is the correction factor and \( D_{ij} \) is the drawdown parameter, which is related to pseudopressure at initial pressure, \( p_{pi} \), and pseudopressure at flowing pressure, \( p_{pwf} \) using Eq. 2.

\[
D_D = \frac{p_{pwf} - p_{ref}}{p_{pwi}} \quad \text{.................................................. (2)}
\]

In this paper, the correction of the slope of the square-root-of-time plot for constant flowing pressure is analytically derived. The method is then validated by comparing its results against test cases that are built using numerical simulation. The effects of initial pressure, flowing pressure, permeability, and formation compressibility are investigated. It is found that in general, the correction factors obtained analytically using the methodology introduced in this paper are more reliable than those obtained using Eq. 1.

Derivation

The base reservoir geometry that is used to analytically develop the correction factor for calculating \( {x_f \sqrt{k}} \) from the slope of the square-root-of-time plot is shown in Fig. 1. This base geometry was chosen because in tight/shale gas reservoirs, it is reasonable to assume that drainage beyond the simulated region is insignificant (Carlson and Mercer 1989; Mayerhofer et al. 2006; Bello and Wattenbarger 2008). It is also assumed that the fracture has infinite conductivity and that there is no skin.

In practice, tight gas and shale gas wells are produced under high drawdown to maximize production. Therefore, the equations presented here are based on the assumption of a constant flowing pressure at the well. Linear-flow theory (Wattenbarger et al. 1998; El-Banbi and Wattenbarger 1998) indicates that at a constant flowing pressure, a plot of 1/\( q \) vs. \( \sqrt{t} \) on Cartesian coordinates is a straight line. The slope of this line can be used to calculate \( {x_f \sqrt{k}} \) for gas as

\[
{x_f \sqrt{k}} = \frac{315.4T}{h \sqrt{(\phi \mu_g c_i)}} \times \frac{1}{p_{pwi} - p_{pwf}} \times \frac{1}{m} \quad \text{................. (3)}
\]

In this equation, \( x_f \) is the fracture half-length; \( k \) is the reservoir permeability; \( T \) is the reservoir temperature; \( h \) is the net-pay thickness; \( \phi \) is the reservoir porosity; \( \mu_g \) is gas viscosity; \( c_i \) is total compressibility (subscript \( i \) refers to initial conditions); \( p_{pwi} \) and \( p_{pwf} \) are pseudopressures at initial pressure and flowing pressure, respectively; and \( m \) is the slope of the 1/\( q \) vs. \( \sqrt{t} \) plot. As mentioned earlier, using this slope in Eq. 3 causes an overestimation of \( {x_f \sqrt{k}} \). This is because Eq. 3 is based on liquid-flow theory and the use of pseudopressure to account for gas. However, this substitution alone is not sufficient. Pseudotime should also have been incorporated to account for changing gas compressibility (Fraim and Wattenbarger 1987; Agarwal et al. 1999; Anderson and Mat- tar 2005), provided that the correct reference pressure is included. In other words, to obtain the correct value of \( {x_f \sqrt{k}} \), the slope of 1/\( q \) vs. \( \sqrt{t} \) should be used in Eq. 3, where \( t_a \) is pseudotime and is defined as follows:

\[
t_a = \left(\frac{\mu_g c_i}{\phi \mu_g c_i}\right)^{\frac{1}{3}} \frac{1}{p_{pwi}^{1/2} c_i^{1/3}} \quad \text{.................................................. (4)}
\]
Here, $\mu_g$ and $\bar{c}_i$ are gas viscosity and total compressibility at the average reservoir pressure, respectively. Traditionally, the average pressure in the whole reservoir is being used to calculate the pseudotime (Fraim and Wattenbarger 1987; Agarwal et al. 1999). However, as indicated by Anderson and Mattar (2005), this introduces serious errors, especially in low-permeability reservoirs. Anderson and Mattar (2005) suggested using the average pressure in the region of influence to calculate pseudotime. The pseudotime calculated using average pressure in the region of influence is called corrected pseudotime (Anderson and Mattar 1999). However, as indicated by Anderson and Mattar (2005), note that Eq. 16a contains the average pressure in the region of influence and initial pressure. Substituting $G_p$ and $G$ from Eq. 6 and Eq. 9, respectively, into Eq. 10 leads to

$$\frac{\bar{p}}{Z''} = \frac{p_i}{Z''} \left(1 - \frac{G_p}{G}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots . \quad (10)$$

This equation shows that for constant-flowing-pressure production, the average pressure in the region of influence is not time dependent during linear flow, and more importantly, it is not initial pressure or average reservoir pressure. Because the average pressure in the region of influence is constant, and by using Eq. 4, the corrected pseudotime, $t''_a$, becomes

$$t''_a = \frac{\bar{p}}{Z''}t \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \quad (12)$$

This means that the corrected pseudotime has a linear relationship with time. Eq. 12 also shows that the slope of the $1/q$ vs. $\sqrt{t''_a}$ plot, $m'$, and the slope of the $1/q$ vs. $\sqrt{t}$ plot, $m$, have the following relationship:

$$m = m' \sqrt{\frac{\bar{p}}{Z''}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \quad (13)$$

As mentioned earlier, in order to obtain the correct value for $x_f\sqrt{k}$ when gas is being analyzed, the slope of the $1/q$ vs. $\sqrt{t''_a}$ plot should be used in Eq. 3. Therefore, using Eq. 13, the following equation can be used to calculate $x_f\sqrt{k}$ from the slope of the $1/q$ vs. $\sqrt{t''_a}$ plot:

$$x_f\sqrt{k} = \frac{315.4T}{h\sqrt{\phi \mu_G \bar{c}_i}} \times \frac{1}{m''} \times \frac{1}{m} \times \sqrt{\frac{\bar{p}}{Z''}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \quad (14)$$

Using Eq. 14, the correction factor $f_{CP}$ that is used to improve the values of $x_f\sqrt{k}$ calculated from the slope of the $1/q$ vs. $\sqrt{t}$ plot can be defined as

$$f_{CP} = \frac{\bar{p}}{Z''} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \quad (15)$$

This equation indicates that the correction factor is related to the average pressure in the region of influence and initial pressure. Substituting $x_f\sqrt{k}$ from Eq. 14 and $B_{gi} = \frac{0.0282ZT}{p_i}$ into Eq. 11 results in

$$\frac{p_i}{Z''} = \frac{1 - 0.281 \left(Z\mu_G \bar{c}_i\right) (p_i - p_{wfi})}{S_{ wp} p_i \sqrt{\bar{p}} \sqrt{\phi \mu_G \bar{c}_i}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \quad (16a)$$

Eq. 16a shows that the average pressure in the region of influence depends on initial pressure, flowing pressure, and gas properties. Eq. 16a can be solved to obtain average pressure in the region of influence, and then the correction factor $f_{CP}$, can be calculated using Eq. 15. To improve $x_f\sqrt{k}$ obtained from linear-flow analysis, $x_f\sqrt{k}$ calculated from Eq. 3 can be multiplied by $f_{CP}$. Note that Eq. 16a contains the average pressure in the region of influence and the gas properties (modified $Z$-factor, viscosity, and total compressibility) at this average pressure as well. To solve this equation, $g(p)$ defined in Eq. 16b is plotted vs. pressure to find the pressure at which $g(p)$ becomes zero:

$$g(p) = \frac{p_i}{Z''} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \quad (16b)$$
Table 1—Input Parameters Used for Numerical Simulation for Different Cases Used to Validate Correction Factors Analytically Derived in This Study

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* The blank cells indicate that the value for that parameter is the same as that of Case 1.

Validity

To validate the correction factor analytically derived in this study, a number of test cases were built for a year of numerically simulated production profile obtained from a black-oil simulator for a single-porosity reservoir. The common parameters among all the test cases are as follows: $T = 120^\circ$F, $h = 100$ ft, $\phi = 10\%$, $S_o = 100\%$, $\gamma_k = 0.65$, $x_f = 250$ ft, $x_c = 500$ ft, and $y_r = 5,000$ ft. The input data for initial pressure, flowing pressure, permeability, and formation compressibility for the numerical-simulation cases are given in Table 1. The blank cells in this table indicate that the value for that parameter is the same as that of Case 1. To model the hydraulic fracture in the numerical simulation, we added a high-permeability grid in the $x$-direction at the center of the reservoir. The permeability of this grid was chosen to be large enough to allow a fracture conductivity of $F_{CFB} = 400$. In other words, the hydraulic fracture was assumed to have infinite conductivity in these simulated cases (i.e., negligible pressure drop along the fracture). Logarithmic gridding was used to model pressure transients accurately. The signatures of expected flow regimes for the hydraulically fractured well shown in Fig. 1 were observed on the semilog derivative plot (not shown here), suggesting the gridding was used was adequate.

First, and for each case, the average pressure in the region of influence was calculated from Eq. 16a and the correction factor was analytically calculated using Eq. 15. Then, $\gamma_k \sqrt{k}$ was calculated using the slope of the $1/q$ vs. $\sqrt{t}$ plot in Eq. 3, and it was compared with the expected value for $\gamma_k \sqrt{k}$ (i.e., entered into numerical simulation). Finally, the expected correction factor was calculated as the ratio of expected $\gamma_k \sqrt{k}$ to calculated $\gamma_k \sqrt{k}$.

The comparison between analytically calculated and expected correction factors for different values of formation compressibilities is shown in Figs. 2 through 4. For $c_f = 0$ and $c_f = 5 \times 10^{-6}$ psi$^{-1}$ (Figs. 2 and 3, respectively), the analytical method presented in this study underestimated the correction factor, whereas Eq. 1 overestimates the correction factor. However, the average of analytically obtained correction factors and those obtained using Eq. 1 agrees well with the expected correction factors. Fig. 4 shows that for $c_f = 5 \times 10^{-6}$ psi$^{-1}$, in general, both methods underestimate the correction factor. However, analytically obtained correction factors are in better agreement with expected correction factors compared with those obtained from Eq. 1. This is because the correlation developed by Ibrahim and Wattenbarger (2005, 2006) does not include the effect of formation compressibility, whereas the analytical method presented in this study considers the effect of formation compressibility through the total compressibility.

Impact of the Distance-of-Investigation Equation

Although we have provided an analytical method for correcting the slope of the $1/q$ vs. $\sqrt{t}$ plot, the new method still leads to underestimating $\gamma_k \sqrt{k}$. In this section, the cause of this underestimation is investigated. According to Eq. 7, the end of linear flow is given by

$$\gamma_k \sqrt{k} = 0.159 \frac{kt_{ef}}{\left(\sqrt{\phi} \mu k c_f \right)}$$

where $\gamma_k$ is reservoir length and $t_{ef}$ is the duration of linear flow. Eq. 17 can be rewritten as follows:

$$t_{ef} = \left[\frac{\gamma_k}{\sqrt{2}} \frac{\left(\sqrt{\phi} \mu k c_f\right)}{2 \times 0.159 \sqrt{k}}\right]$$

Case 5 (where $p_i = 10,000$ psi, $p_{wf} = 3,000$ psi, $k = 0.1$ md, and $c_f = 0$) is chosen to evaluate Eq. 18. The end of linear flow is estimated to be 313 days for Case 5 using Eq. 18. Fig. 5 shows the pressure distribution in the reservoir for this case after 313 days. Although Eq. 18 predicts that the end of linear flow (or start of boundary-dominated flow) is after 313 days, this figure clearly indicates that the pressure propagation reaches the boundaries before 313 days. Fig. 6 shows the semilog derivative vs. time plotted on log-log coordinates. This figure also shows that the linear flow ended before 313 days in Case 5. Therefore, there is a possibility that Eq. 7 underestimates the distance of investigation, which results in underestimating the average pressure in the region of influence. This can explain underestimating the correction factor using the method introduced in this study.

To evaluate Eq. 7 for calculating the distance of investigation, a number of test cases were built using numerical simulation. The input data for initial pressure, flowing pressure, and permeability for these cases are given in Table 2. The formation compressibility is zero, and the other parameters not listed in Table 2 are the same as in previously presented cases. First, and for each case, the end of linear flow was calculated using Eq. 18. Then, the end of linear flow was obtained using the pressure-distribution profile in the reservoir. The end of linear flow in this method is defined as the time at which the pressure drop at the upper and lower reservoir boundaries reaches 10% of the maximum pressure drop (i.e., $\Delta p = 0.1 \left(p_i - p_{wf}\right)$). For example, for $p_i = 10,000$ psi and $p_{wf} = 3,000$ psi, the end of linear flow is assumed to be reached when the pressure at the boundaries reaches 9,300 psi. Fig. 7 shows a plot of the end of linear flow, obtained using the pressure profile in the reservoir, $(\gamma_k \sqrt{k})$, vs. the end of linear flow calculated
using Eq. 18, \((t_{el})_c\). On the basis of this data, \((t_{el})_o\) is correlated to \((t_{el})_c\) by applying the linear regression

\[
(t_{el})_o = 0.613 (t_{el})_c. \quad \ldots \quad (19)
\]

Using Eq. 19, the following equation can be obtained to calculate the distance of investigation:

\[
y_{10\%} = 0.203 \sqrt{\frac{kt}{\phi \mu_k c_f}}, \quad \ldots \quad (20)
\]

where \(y_{10\%}\) is the distance at which pressure drop is 10% of the maximum pressure drop. Using Eq. 20 instead of Eq. 7 for derivation presented in the previous sections, Eq. 16a will be changed to the following equation:

**Fig. 2**—Comparison between calculated correction factors calculated from this study (combination of Eqs. 15 and 16a) and Ibrahim and Wattenbarger (2005, 2006) method with expected correction factor for \(c_f = 0\).

**Fig. 3**—Comparison between calculated correction factors calculated from this study (combination of Eqs. 15 and 16a) and Ibrahim and Wattenbarger (2005, 2006) method with expected correction factor for \(c_f = 5 \times 10^{-6} \text{ psi}^{-1}\).
The comparison between analytically calculated correction factors using Eq. 21 instead of Eq. 16a and expected correction factors for different values of formation compressibilities is shown in Figs. 8 through 10. It can be seen that using Eq. 21 significantly improves the analytically calculated correction factors.

**Approximate Solution**

In this section, the simplified form of Eq. 21 is obtained using the following assumptions:

1. The gas is ideal ($Z = 1$). Using the definition of gas compressibility, this assumption leads to

$$
\frac{p}{Z_i} = \frac{p_i}{Z_i} \left[ 1 - 0.220 \frac{\left(Z \mu_g c_i\right)}{\left(p_i - p_{wir}\right)} \sqrt{\mu_g c_i} \right] \quad \ldots \quad (21)
$$

The comparison between analytically calculated correction factors using Eq. 21 instead of Eq. 16a and expected correction factors for different values of formation compressibilities is shown in Figs. 8 through 10. It can be seen that using Eq. 21 significantly improves the analytically calculated correction factors.
2. Gas viscosity is not changing with pressure. Using the definition of pseudopressure,

\[ p_{pi} = \frac{2}{l_{gi}} \int p \, dp = \frac{p_t^2}{l_{gi}} \]  \hspace{1cm} (23)

3. Total compressibility is dominated by gas compressibility, that is,

\[ c_r = S_g c_g. \] \hspace{1cm} (24)

Eq. 25 is the simplified form of Eq. 21, which is obtained by combining Eqs. 21 through 24 as follows:

\[ \bar{p} = p_i \left( 1 - 0.220 \frac{p_{pi} - p_{wrf}}{p_{pi}} \sqrt{\frac{p_r}{\bar{p}}} \right). \] \hspace{1cm} (25)

Because gas viscosity is assumed to be constant and gas compressibility is assumed to be inversely proportional to pressure, Eq. 15 simplifies to

\[ f_{CP} = \sqrt{\frac{\mu_g c_t}{\mu_i c_t}} = \sqrt{\frac{c_i}{c_t}} = \sqrt{\frac{p}{p_i}}. \] \hspace{1cm} (26)

Combining Eqs. 25 and 26, we will end up with the following equation:

\[ f_{CP}^3 - f_{CP} + 0.220 D_D = 0. \] \hspace{1cm} (27)

where \( D_D \) is the drawdown parameter defined in Eq. 2.

Eq. 27 shows that under the assumptions presented previously, the correction factor, \( f_{CP} \), depends only on the drawdown parameter defined by Ibrahim and Wattenbarger (2005, 2006). Fig. 11 shows the comparison between the correction factors obtained from Eq. 1 and Eq. 27. This figure shows that the correction factors obtained using Eq. 27 and the empirical values calculated from the correlation developed by Ibrahim and Wattenbarger (2005, 2006) are very close for an ideal gas with constant viscosity when gas compressibility dominates the total compressibility.

### Differences Between Constant-Flowing-Pressure and Constant-Rate-Production Linear Flow

Nobakht and Clarkson (2012) presented a method to analyze linear flow in gas reservoirs producing with constant-rate production. Comparing the findings in that work and those in this study, the following differences are observed between constant-flowing-pressure and constant-rate-production linear flow in gas reservoirs:

1. The average pressure in the region of influence is constant (Eq. 11) for constant flowing pressure, whereas the average pressure in the region of influence is time dependent and decreases with time for constant-rate production.

2. The corrected pseudotime has a linear relationship with time (Eq. 12) for constant-flowing-pressure production. However, for constant-rate production, corrected pseudotime is almost equal

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**Table 2—Input Parameters Used for Numerical Simulation for Different Cases to Evaluate Eq. 7**

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</tbody>
</table>
3. For constant-flowing-pressure production, the square-root-of-time plot is a straight line; however, for constant-rate production, the square-root-of-time plot may not be a straight line if corrected pseudotime is not used, and its shape depends on gas-production rate.

4. For constant-rate production, the proposed analysis method is iterative, whereas for constant-flowing-pressure production, the
procedure presented is not iterative and it involves only finding a correction factor.

**Application to Multifractured Horizontal Wells Completed in Ultralow-Permeability Reservoirs**

Multifractured horizontal wells are the most commonly used method for exploiting shale gas reservoirs. Because massive hydraulic fractures are typically created, the dominant flow regime observed in these wells is linear flow to fractures that may last for a long time because of extremely low permeability of shale gas reservoirs. Therefore, it is of practical interest to validate the methodology presented in this study for application in multifractured horizontal wells.

Linear-flow analysis results in the product of total fracture length (or surface area to flow) and square root of permeability. If
the permeability is known, the total fracture length can be obtained. This means that when analyzing linear flow for a hydraulically fractured vertical well, the length of the fracture can be obtained, whereas linear-flow analysis for a multifractured horizontal well results in total fracture length (i.e., the summation of lengths of all fractures). This is essentially the difference between analyzing linear flow for hydraulically fractured vertical wells and horizontal wells with multiple fractures. On the basis of this discussion, one can plot inverse gas rate vs. square root of time for a multifractured horizontal well and use the slope of this plot in Eq. 3 to calculate the product of total fracture length and square root of permeability. Then, this value can be multiplied by the correction factor calculated using the method presented in this study or that calculated from Eq. 1.

The permeability for the test cases presented in Table 1 is more appropriate for tight gas reservoirs. To validate the applicability of the method presented in this study for shale gas reservoirs, a new test case was built similar to Case 24 in Table 1, except \( c_f = 0 \) and the permeability was reduced to \( k = 100 \, \text{nd} \). Using the slope of inverse gas rate vs. the square root of time plot (not shown here) in Eq. 3 and \( k = 100 \, \text{nd} \), fracture half-length was calculated to be 325 ft. The correction factor obtained using the method presented in this study (i.e., Eq. 15) for this case was 0.77. Therefore, the improved fracture half-length was \( 0.77 \times 325 = 250 \, \text{ft} \), which was the same as the expected value of 250 ft (i.e., the input to numerical simulation). This demonstrates the validity of the method presented in this study in permeability range for shale gas reservoirs. For comparison, the correction factor using the Ibrahim and Wattenbarger (2005, 2006) method (calculated from Eq. 1) was 0.83, which corresponds to a fracture half-length of 270 ft.

**Discussion**

The derivation presented in this study is developed assuming infinite-conductivity fracture(s) and no skin. However, as mentioned by Nobakht and Mattar (2012), shale gas production often exhibits linear flow with a significant apparent skin. The apparent skin can be caused by flow convergence in a horizontal well (Bello and Wattenbarger 2010), finite conductivity of the fractures (Anderson et al. 2010), and/or multiphase flow in the reservoir (Nobakht and Mattar 2012). As mentioned by Nobakht and Mattar (2012), all these effects create an extra pressure drop, which for all practical purposes, can be treated as an apparent skin effect.

The methodology presented in this study can be applied when dealing with fractures with finite conductivity. In other words, \( x_f \sqrt{k} \) calculated from Eq. 3 can be multiplied by the correction factor to improve the linear-flow analysis. To verify this, a simulation case was created for the multifractured horizontal well shown in Fig. 12. The reservoir permeability was assumed to be 100 nd, the length of the horizontal well was 2,500 ft, and there were five hydraulic fractures, each of which had a 520-ft half-length and dimensionless fracture conductivity \( F_{CD} \) of 50. A simulated gas-rate profile was created using real flowing-pressure data obtained from a multifractured horizontal well that was producing under high drawdown. Fig. 13 shows a plot of inverse gas rate vs. square root of time for this case. Using the slope of this line in Eq. 3 and \( k = 100 \, \text{nd} \), the total fracture half-length becomes 3,485 ft. Therefore, the half-lengths of individual fractures \( y_f \) were calculated to be 697 ft, which is 35% greater than the expected value of 520 ft (i.e., input to simulation). Using the correction factor calculated using the method presented in this study (i.e., Eq. 15) for this case was 0.77. Therefore, the improved fracture half-length was \( 0.77 \times 697 = 520 \, \text{ft} \), which was the same as the expected value of 520 ft (i.e., the input to numerical simulation).
factor presented in this study improves \( y_f \) to 540 ft. It should be mentioned that if we use the correction factor from Eq. 1, \( y_f \) becomes 585 ft. Fig. 13 shows that the intercept of the line is positive, which is an indication of apparent skin (Nobakht and Mattar 2012), caused in this case by finite conductivity of fractures.

**Field Examples**

Case 1. This case study is for a multifractured horizontal well drilled in a Barnett-shale gas reservoir with \( p_i = 3,100 \) psi, \( T_R = 106^\circ\text{F} \), \( h = 260 \) ft, \( \phi = 4\% \), \( S_{yi} = 90\% \), \( S_{wi} = 10\% \), and \( c_f = 7 \times 10^{-6} \) psi\(^{-1} \). Fig. 14 shows the drawdown parameter, \( D_D \), vs. time for
this well. This figure shows that the well is producing under high-drawdown condition, and therefore the methodology presented in this study for constant flowing pressure can be applied. Fig. 15 shows the inverse gas rate vs. the square root of time plot for this case. Clearly, the data form a straight line that indicates that linear flow is the dominant flow regime. Using the slope of the plot in Eq. 3 and assuming \( k = 1000 \text{ nd} \), the total fracture half-length is calculated to be 3,290 ft. For this case, the correction factor calculated from Eq. 1 [i.e., Ibrahim and Wattenbarger (2005, 2006)] is approximately 0.84 and the correction factor calculated from the method presented in this study (i.e., combination of Eqs. 15 and 21) is approximately 0.86. Therefore, the total fracture half-lengths calculated using these two methods are 2,760 and 2,830 ft, respectively. In this example, the correction factors calculated from Eq. 1 and Eq. 15 are very close.

**Case 2.** The variation of drawdown parameter with time for this multifractured horizontal well is shown in Fig. 16. This figure indicates that, for the most part, this well is producing under high-drawdown condition, and therefore the methodology presented in this study for constant flowing pressure can be applied. Fig. 17 shows the inverse gas rate vs. the square root of time plot for this well. Clearly, the data form a straight line, indicating that linear flow is the dominant flow regime. Using the slope of the plot in Eq. 3 and assuming \( k = 100 \text{ nd} \), the total fracture half-length is calculated to be 2,570 ft. For this case, the correction factors calculated from Eq. 1 and Eq. 15 are approximately 0.85 and 0.74, respectively. Therefore, the total fracture half-lengths calculated using these two methods are 2,180 and 1,900 ft, respectively.

**Case 3.** This case, which is a 5,000-ft horizontal well with 10 equally spaced fractures along the horizontal well, is presented to show the applicability of the method presented in this study to real data when dealing with finite-conductivity fractures. The properties of this reservoir (Marcellus-shale gas) are as follows: \( p_i = 5,300 \text{ psi} \), \( T_R = 150^\circ \text{F} \), \( h = 100 \text{ ft} \), \( \phi = 8\% \), \( S_{wi} = 76\% \), \( S_{wi} = 24\% \), and \( c_f = 5 \times 10^{-6} \text{ psi}^{-1} \). The plot of drawdown parameter vs. time for this well is shown in Fig. 18. This figure shows that, as with Case 1 and Case 2, this well is also producing under high drawdown (between 95 and 100%). Fig. 19 shows the inverse gas rate vs. the square root of time plot for this well. The data form a straight line, indicating existence of linear flow throughout the production to date. This figure also shows that the intercept of the line is positive because of a significant amount of skin (Nobakht and Mattar 2012). In the next step, a numerical model is used to history match the production data. Because linear flow was the only regime available, we considered only the stimulated reservoir volume for modeling (i.e., schematic similar to Fig. 12 with 10 fractures). Fig. 20 shows the comparison between rates obtained from numerical simulation and historical data. The matrix permeability, dimensionless fracture conductivity, and half-length of each fracture is estimated to be \( k = 90 \text{ nd} \), \( FCD = 25 \), and \( y_f = 520 \text{ ft} \), respectively.

From the slope of the line in Fig. 19, the product of the total fracture half-length and the square root of the reservoir (or matrix) permeability is estimated to be 63,562 md\(^{1/2} \text{ ft} \) from Eq. 3, which corresponds to \( y_f \sqrt{k} \) of 6.3562 md\(^{1/2} \text{ ft} \). Using \( k = 90 \text{ nd} \) (obtained from history matching), \( y_f \) is calculated to be 670 ft, which is almost 30% greater than the value obtained from numerical simulation. Applying the correction factor presented in this study, the half-lengths of each of the fractures become 529 ft, which agrees very well with the data obtained from numerical simulation. Using the correction factor from Eq. 1, the improved fracture half-length becomes 565 ft.

**Conclusions**

This paper presented an analytical method to determine a correction factor for calculating \( y_f \sqrt{k} \) from the slope of inverse gas rate vs. the square root of time plot for constant-flowing-pressure production. It was shown that for the reservoir geometry shown in Fig. 1, the average pressure in the region of influence is constant.

**Fig. 15—Inverse gas rate vs. the square root of time plot for Field Example 1.**
during linear flow. The method was then validated by comparing its results against numerically simulated cases. It was found that, in general, the correction factors obtained using the method proposed in this study are lower than those expected. It was also found that Eq. 7 underestimates distance of investigation for linear flow. This equation was then modified to match the distance of investigation observed in simulation. Using the modified equation for distance of investigation, the analytically calculated
correction factors were in good agreement with those expected. Then, it was shown that for a case of ideal gas with constant viscosity, if total compressibility is dominated by gas compressibility, the correction factor depends only on the drawdown parameter defined in Eq. 2. Finally, the application of the method to multifractured horizontal wells was discussed, and the method was applied to three multifractured horizontal wells drilled in different shale plays.

Fig. 18—Drawdown parameter vs. time plot for Field Example 3.

Fig. 19—Inverse gas rate vs. the square root of time plot for Field Example 3.
Nomenclature

\[ A = \text{area of region of influence, ft}^2 \]
\[ B_{gi} = \text{initial gas formation volume factor, ft}^3/\text{scf} \]
\[ c_f = \text{formation compressibility, psi}^{-1} \]
\[ c_g = \text{gas compressibility, psi}^{-1} \]
\[ c_t = \text{total compressibility, psi}^{-1} \]
\[ D_{1p} = \text{drawdown parameter defined in Eq. 2, fraction} \]
\[ f_{CP} = \text{correction factor} \]
\[ F_{CD} = \text{dimensionless fracture conductivity, dimensionless} \]
\[ G = \text{gas in place in the region of influence, scf} \]
\[ G_p = \text{cumulative gas production, scf} \]
\[ h = \text{net-pay thickness, ft} \]
\[ k = \text{permeability, md} \]
\[ L_e = \text{horizontal-well length, ft} \]
\[ m = \text{slope of the } 1/q \text{ vs. } \sqrt{t} \text{ plot, day}^{1/2}/\text{Mscf} \]
\[ m' = \text{slope of the } 1/q \text{ vs. } \sqrt{t_1} \text{ plot, day}^{1/2}/\text{Mscf} \]
\[ p_i = \text{initial pressure, psi} \]
\[ p_i^p = \text{pseudopressure at initial pressure, psi}^{2/3} \text{cp} \]
\[ p_{pwf} = \text{pseudopressure at flowing pressure, psi}^{2/3} \text{cp} \]
\[ q = \text{gas rate, Mscf/D} \]
\[ S_g = \text{initial gas saturation, fraction} \]
\[ S_wi = \text{initial water saturation, fraction} \]
\[ t = \text{time, days} \]
\[ t_p = \text{pseudotime, days} \]
\[ t_{1p} = \text{corrected pseudotime, days} \]
\[ t_{oLF} = \text{end of linear flow, days} \]
\[ t_{oLF}_{C} = \text{end of linear flow calculated using Eq. 18, days} \]
\[ t_{oLF}_{D} = \text{end of linear flow obtained using the pressure profile in the reservoir, days} \]
\[ T = \text{reservoir temperature, } ^\circ\text{R} \]
\[ x_e = \text{reservoir width, ft} \]
\[ x_f = \text{fracture half-length in x-direction, ft} \]
\[ y = \text{distance of investigation, ft} \]
\[ y_{10\%} = \text{distance at which pressure drop is 10\% of the maximum pressure drop, ft} \]
\[ y_e = \text{reservoir length, ft} \]
\[ y_f = \text{fracture half-length in y-direction, ft} \]
\[ Z = \text{gas-compressibility factor} \]
\[ \gamma_g = \text{reservoir gas specific gravity (air } = 1) \]
\[ \mu_g = \text{gas viscosity, cp} \]
\[ \phi = \text{porosity, fraction} \]

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